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Tugas 1 Persamaan Diferensial Parsial (F)

2. Tentukan persamaan differensial parsial yang timbul dari masing-masing persamaan berikut :

a. $z = x + y + f(xy)$

$\Rightarrow \frac{\partial z}{\partial x} = 1 + y f'(xy)$

$\Rightarrow \frac{\partial z}{\partial y} = 1 + x f'(xy)$

misal $\frac{\partial z}{\partial x} = p$; $\frac{\partial z}{\partial y} = q$

$\Rightarrow p = 1 + y f'(xy)$ $\qquad \Rightarrow q = 1 + x f'(xy)$

$\frac{p-1}{y} = f'(xy)$ .... ① $\qquad \frac{q-1}{x} = f'(xy)$ ... ②

Substitusi ① ke ②

$\frac{p-1}{y} = f'(xy) \longrightarrow \frac{p-1}{y} = \frac{q-1}{x}$

$x(p-1) = y(q-1)$

Jadi, PD yang timbul adalah :

$$x\left(\frac{\partial z}{\partial x} - 1\right) = y\left(\frac{\partial z}{\partial y} - 1\right)$$

d). $2z = (\alpha x + y)^2 + \beta$

$z = \frac{1}{2}\left[(\alpha x + y)^2 + \beta\right]$

$\frac{\partial z}{\partial x} = \frac{1}{2}\left[2(\alpha x + y).\alpha\right]$ $\qquad \frac{\partial z}{\partial y} = \frac{1}{2}\left[2(\alpha x + y)(1)\right]$

$= \alpha(\alpha x + y)$ $\qquad = (\alpha x + y)$

Misal : $\frac{\partial z}{\partial x} = p$ $\qquad \frac{\partial z}{\partial y} = q$

$p = \alpha(\alpha x + y)$ ... ① $\qquad q = \alpha x + y$ ... ②

Substitusi ② ke ①

$p = \alpha(\alpha x + y)$

$= \alpha q$

$\alpha = \frac{p}{q}$ .... ③

Dari persamaan awal dan ②, diperoleh hubungan:

$2z = (\alpha x + y)^2 + \beta$

$2z = q^2 + \beta \Rightarrow \beta = 2z - q^2$

Didapatkan :

$2z = (\alpha x + y)^2 + \beta$

$\cancel{2z} = \left(\frac{p}{q}x + y\right)^2 + \cancel{2z} - q^2$

$q^2 = \left(\frac{p}{q}x + y\right)^2$

$q = \frac{p}{q}x + y$

$q = \frac{px + qy}{q}$

$q^2 = px + qy$

Jadi, PD yang timbul adalah

$$\left[\frac{\partial z}{\partial y}\right]^2 = \frac{\partial z}{\partial x}x + \frac{\partial z}{\partial y}y$$

3). Dapatkan penyelesaian PDP yang diberikan oleh persamaan berikut dengan menggunakan integral secara langsung atau pemisahan variabel.

f). $U_x = y U_y$

misal : $U(x,y) = X(x).Y(y) = XY$

$U_x = y U_y$

$\dot{X}Y = y X\dot{Y}$

$\frac{\dot{X}}{X} = y\frac{\dot{Y}}{Y} = \lambda$

※ $\frac{\dot{X}}{X} = \lambda$

$\int \frac{dx}{x} = \int \lambda\, dx$

$\ln x = \lambda x + C_1$

$x = e^{\lambda x + C_1}$

$X = C_1 e^{\lambda x}$

※ $y.\frac{\dot{Y}}{Y} = \lambda$

$y.\frac{dy}{Y dy} = \lambda$

$\int \frac{dY}{Y} = \int \frac{\lambda}{y} dy$

$\ln Y = \lambda \ln y + C_2$

$Y = e^{\lambda \ln y + C_2}$

$Y = C_2 y^\lambda$

PUPD : $U(x,y) = XY$

$$= C_1 e^{\lambda x} . C_2 y^{\lambda}$$
$$= C e^{\lambda x} y^{\lambda}$$
$$= C(ye^{x})^{\lambda}$$

Jadi, PUPD nya adalah $C(ye^{x})^{\lambda}$

g). $U_x + U_y = 2(x+y)$

$U_x = \frac{\partial u}{\partial x} = \dot{X}Y$

$U_y = \frac{\partial u}{\partial y} = X\dot{Y}$

$\dot{X}Y + X\dot{Y} = 2(x+y)$

$\dot{X}Y - 2y = 2x - X\dot{Y}$

$\frac{\dot{x}}{x} - \frac{2x}{xy} = \frac{2y}{xy} - \frac{\dot{y}}{y} = \lambda$

$\frac{\dot{x}}{x} - \frac{2}{y} = \frac{2}{x} - \frac{\dot{y}}{y} = \lambda$

$\frac{\dot{x}}{x} - \frac{2}{x} = \frac{2}{y} - \frac{\dot{y}}{y} = \lambda$

$\Rightarrow \frac{\dot{x}}{x} = \lambda + \frac{2}{x}$

$\int \frac{dx}{x} = \int \lambda + \frac{2}{x}\, dx$

$\ln x = \lambda x + 2\ln x + C$

$x = e^{\lambda x + 2\ln x + C}$

$x = Ce^{\lambda x + 2\ln x}$

$x = C_1 e^{\lambda x + \ln x^2}$

$\Rightarrow \frac{\dot{y}}{y} = \frac{-2}{y} - \lambda$

$\int \frac{dy}{y} = \int -\frac{2}{y} - \lambda\, dy$

$\ln y = -2\ln y - \lambda y + C$

$e^{\ln y} = e^{-\ln y^2 - \lambda y + C}$

$y = C_2 (e^{-\lambda y}) \frac{1}{y^2}$

$U = XY = C_1 e^{\ln x^2 + \lambda x} . C_2 e^{-\lambda y} . \frac{1}{y^2}$

$= Ce^{\ln \lambda(x-y)} . \frac{x^2}{y^2}$

$= C\left(\frac{x}{y}\right)^2 e^{\lambda(x-y)}$

3 h). $U_x = U_y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$$

misal : $U = XY$

$$\frac{\partial u}{\partial x} = \dot{X}Y \qquad \frac{\partial u}{\partial y} = X\dot{Y}$$

$$\dot{X}Y = X\dot{Y} \quad : xy$$

$$\frac{\dot{x}}{x} = \frac{\dot{y}}{y} = \lambda$$

$\cdot\!> \frac{\dot{x}}{x} = \lambda$

$$\int \frac{dx}{x} = \int \lambda\, dx$$

$$\ln x = \lambda x + C$$

$$e^{\ln x} = e^{\lambda x + C}$$

$$x = C_1 e^{\lambda x}$$

$\cdot\!> \frac{\dot{y}}{y} = \lambda$

$$\int \frac{dy}{y} = \int \lambda\, dy$$

$$\ln y = \lambda y + C$$

$$e^{\ln y} = e^{\lambda y + C}$$

$$y = C_2 e^{\lambda y}$$

PUPD : $U(x,y) = xy$

$$= C_1 e^{\lambda x} C_2 e^{\lambda y}$$

$$= C e^{\lambda(x+y)}$$

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i). $ay\, U_x = bx\, U_y$, a, b konstan

$$\frac{1}{bx} U_x = \frac{1}{ay} U_y$$

$$\frac{1}{bx} \dot{x}y = \frac{1}{ay} x\dot{y}$$

$$\frac{1}{bx} \cdot \frac{\dot{x}}{x} = \frac{1}{ay} \frac{\dot{y}}{y} = \lambda$$

$\cdot\!> \frac{\dot{x}}{x} = \lambda bx$

$$\int \frac{dx}{x} = \int \lambda bx\, dx$$

$$\ln x = \lambda b \cdot \tfrac{1}{2}x^2 + C$$

$$e^{\ln x} = e^{\lambda \frac{b}{2} x^2 + C}$$

$$x = e^{\frac{\lambda b x^2}{2} + C}$$

$$x = C_1 e^{\frac{\lambda b x^2}{2}}$$

$\cdot\!> \frac{\dot{y}}{y} = \lambda ay$

$$\int \frac{dy}{y} = \int \lambda ay\, dy$$

$$\ln y = \frac{\lambda a y^2}{2} + C$$

$$y = e^{\frac{\lambda a y^2}{2} + C}$$

$$y = C_2 e^{\frac{\lambda a y^2}{2}}$$

PUPD = $xy = C_1 e^{\frac{\lambda b x^2}{2}} C_2 e^{\frac{\lambda a y^2}{2}}$

$$= C e^{\frac{\lambda}{2}(bx^2 + ay^2)}$$

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j). $xU_x + yU_y = 0$

$$x \cdot \dot{X}y = -y X\dot{y}$$

$$x \cdot \frac{\dot{x}}{x} = -y \frac{\dot{y}}{y} = \lambda$$

$\cdot\!> x \cdot \frac{\dot{x}}{x} = \lambda$

$$\int \frac{dx}{x\, dx} = \int \frac{\lambda}{x}\, dx$$

$$\ln x = \lambda \ln x + C$$

$$e^{\ln x} = e^{\lambda \ln x + C}$$

$$x = e^{\lambda \ln x + C_1}$$

$$= C_1 x^{\lambda}$$

$\cdot\!> -y \frac{\dot{y}}{y} = \lambda$

$$\int \frac{dy}{y} = \int \frac{\lambda}{y}\, dy$$

$$\ln y = -\lambda \ln y + C$$

$$\cancel{\ln y}\; e^{\ln y} = e^{-\lambda \ln y + C}$$

$$y = e^{-\lambda \ln y + C}$$

$$y = C_2$$

PUPD : XY

$$= C_1 x^{\lambda} C_2 y^{-\lambda}$$

$$= C x^{\lambda} y^{-\lambda}$$

$$= C \left(\frac{x}{y}\right)^{\lambda}$$

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4). a). $U_x + u = u_y$, $u(x,0) = 4e^{-3x}$

$$\dot{X}Y + XY = X\dot{Y}$$

$$\frac{\dot{X}}{X} + 1 = \frac{\dot{Y}}{Y} \longrightarrow \frac{\dot{X}}{X} + 1 - \frac{\dot{Y}}{Y} = \lambda$$

$\cdot\!> \frac{\dot{X}}{x} = \lambda - 1$

$$\int \frac{dx}{x} = \int (\lambda - 1)\, dx$$

$$\ln x = x(\lambda - 1) + C$$

$$e^{\ln x} = e^{x(\lambda - 1) + C}$$

$$x = e^{\lambda x - x + C}$$

$$x = C_1 e^{\lambda x - x}$$

$\cdot\!> \frac{\dot{Y}}{y} = \lambda$

$$\int -\frac{dy}{y} = \int \lambda\, dy$$

$$-\ln y = \lambda y + C$$

$$\ln y = -\lambda y + C$$

$$e^{\ln y} = e^{-\lambda y + C}$$

$$y = Ce^{-\lambda y}$$

PUPD = xy

$$= C_1 e^{\lambda x - x} C_2 e^{-\lambda y}$$

$$= C e^{\lambda x - x - \lambda y}$$

$$= C e^{\lambda(x-y) - x}$$

$$u(x,0) = Ce^{\lambda x - x} = 4e^{-3x}$$

$$Ce^{(\lambda - 1)x} = 4e^{-3x}$$

$$\Longrightarrow \quad C = 4 , \quad \lambda - 1 = -3$$

$$\lambda = -2$$

Penyelesaian Khusus : $4e^{-2x+2y-x}$

$$U = 4e^{-3x+2y}$$

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c). $U_x + 2U_y = 0$; $U(0,y) = 3e^{-2y}$

$\dot{X}Y + 2X\dot{Y} = 0$

$\dot{X}Y = -2X\dot{Y}$

$\frac{\dot{X}}{X} = -2\frac{\dot{Y}}{Y} = \lambda$

-> $\frac{\dot{X}}{X} = \lambda$ -> $-2\frac{\dot{Y}}{Y} = \lambda$

$\int \frac{1}{X}dx = \int \lambda\, dx$ $\int -\frac{2}{Y}dy = \int \lambda\, dy$

$\ln X = \lambda x + C$ $-2\ln Y = \lambda y + C$

$X = e^{\lambda x + c}$ $\ln Y = \frac{\lambda y}{-2} + C$

$X = C_1 e^{\lambda x}$ $e^{\ln Y} = e^{-\lambda y/2 + C}$

$Y = C e^{-\frac{\lambda y}{2}}$

PUPD : $XY = C_1 e^{\lambda x} \cdot C_2 e^{-\frac{\lambda y}{2}}$

$= Ce^{\lambda(x - \frac{y}{2})}$

Substitusi $U(0,y) = 3e^{-2y}$

$Ce^{\lambda(-\frac{y}{2})} = 3e^{-2y}$

※ $C = 3$ $\frac{\lambda}{2} = 2 \rightarrow \lambda = 4$

Penyelesaian khusus: $U = 3e^{4(x - \frac{y}{2})}$ //

e). $x^2 u_{xy} + 9y^2 u = 0$; $u(x,0) = e^{1/x}$

$U = XY$ -> $U_x = \dot{X}Y$ -> $U_y = X\dot{Y}$

$x^2(\dot{X}\dot{Y}) + 9y^2(XY) = 0$

$x^2(\dot{X}\dot{Y}) = -9y^2(XY) = \lambda$

-> $\frac{x^2(\dot{X})}{X} = \frac{-9y^2 \cdot Y}{\dot{Y}} = \lambda$

\# $\int \frac{x^2}{X}dx = \int \lambda dx$

$\int \frac{dx}{X} = \int \frac{\lambda}{x^2}dx$

$\ln X = -\frac{\lambda}{x} + C$

$e^{\ln X} = e^{-\lambda/x + C}$

$X = Ce^{-\lambda/x}$

* $\frac{-9y^2 Y}{\dot{Y}} = \lambda$

$-9y^2 dy = \frac{\lambda}{Y}dy$

$-9\int y^2 dy = \lambda \int \frac{1}{y}dy$

$-3y^3 + C_2 = \lambda \ln y$

$-\frac{3}{\lambda}y^3 + C_2 = \ln y$

$e^{-\frac{3}{\lambda}y^3 + C_2} = e^{\ln y}$

$y = Ce^{-\frac{3}{\lambda}y^3}$

PUPD : $U(x,y) = XY$

$= Ce^{-\lambda/x}\, ce^{-(3/\lambda)y^3}$

$= Ce^{-\frac{\lambda}{x} - \frac{3}{\lambda}y^3}$ //

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